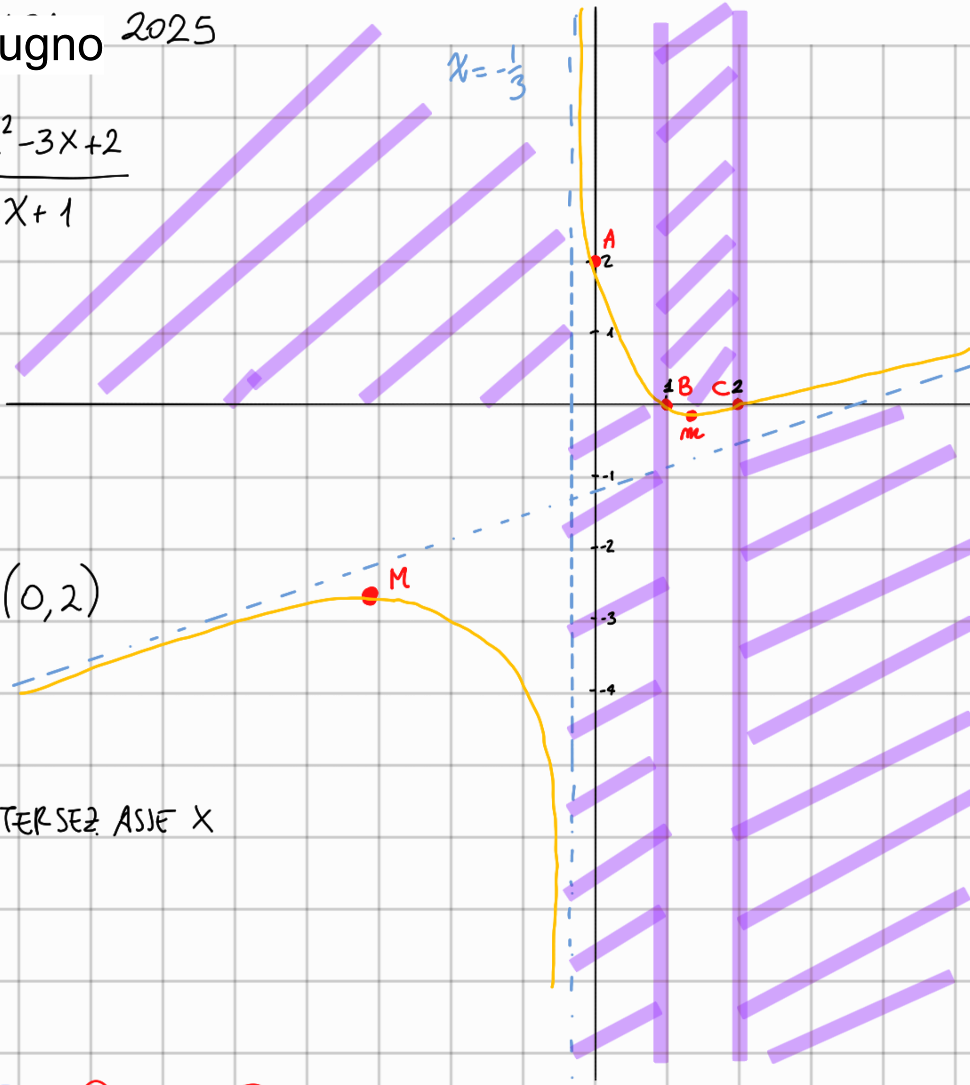
SOLUZIONE ESAME 16 Giugno 2025

$\boxed{\text{ESERCIZIO 1}}$ a. $f(x) = \dfrac{x^2-3x+2}{3x+1}$



■ DOMINIO: $3x+1 \neq 0 \rightarrow x \neq -\frac{1}{3}$

$$D = \left(-\infty, -\frac{1}{3}\right) \cup \left(-\frac{1}{3}, +\infty\right)$$

■ INTERSEZIONE ASSE Y

$$f(0) = \frac{0^2-3\cdot 0+2}{3\cdot 0+1} = \frac{2}{1} = 2$$

$A = (0,2)$

■ STUDIO DEL SEGNO E INTERSEZ. ASSE X

$$\frac{x^2-3x+2}{3x+1} \geq 0$$

NUM

$x^2-3x+2 \geq 0$

$\Delta = 9-8 = 1$

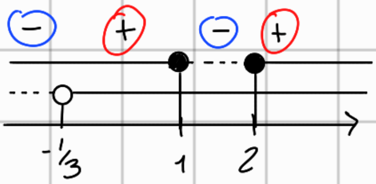
$x_{1,2} = \dfrac{3 \pm \sqrt{1}}{2}$ $\quad x_1 = 1 \quad x_2 = 2$

$x \leq 1 \vee x \geq 2$

DEN

$3x+1 > 0$

$x > -\dfrac{1}{3}$



$\rightarrow$ f è POSITIVA per $x \in \left(-\frac{1}{3}, 1\right) \cup (2, +\infty)$

f è NEGATIVA per $x \in \left(-\infty, -\frac{1}{3}\right) \cup (1,2)$

f è NULLA per $x = 1 \vee x = 2$

$B(1,0)$, $C(2,0)$

■ LIMITI AGLI ESTREMI DEL DOMINIO

$$\lim_{x \to -\frac{1}{3}^-} \frac{x^2-3x+2}{\underset{(-1)^-+1}{3x+1}} = \frac{\overset{+}{\frac{1}{9}+1+2}}{\underset{1^-+1}{0^-}} = -\infty$$

$$\lim_{x \to -\frac{1}{3}^+} \frac{x^2-3x+2}{3x+1} = \frac{\oplus}{0^+} = +\infty$$

$\boxed{x = -1/3}$ A. V. COMPLETO

$$\lim_{x \to -\infty} \frac{x^2-3x+2 \xrightarrow{} +\infty}{\underset{\downarrow -\infty}{3x+1}} = -\infty$$

F.I.

PER CONFRONTO DI INFINITI

$$\lim_{x \to +\infty} \frac{x^2-3x+2}{3x+1} = +\infty$$

NO A. ORIZZONTALE

CONTROLLO A. OBLIQUO

$$\lim_{x\to\pm\infty} \frac{x^2-3x+2}{3x+1}\cdot\frac{1}{x} = \lim_{x\to\pm\infty}\frac{x^2-3x+2}{3x^2+1} = \frac{1}{3} = m$$

$$\lim_{x\to\pm\infty} \frac{x^2-3x+2}{3x-1} - \frac{1}{3}x = \lim_{x\to\pm\infty}\frac{\cancel{3x^2}-9x+6-\cancel{3x^2}-x}{3(3x+1)} = \lim_{x\to\pm\infty}\frac{-10x+6}{9x+1} = -\frac{10}{9} = q$$

ASINTOTO OBLIQUO

$$y = \frac{1}{3}x - \frac{10}{9}$$

DERIVATA PRIMA, MASSIMI E MINIMI

$$f'(x) = \frac{(2x-3)(3x+1)-(x^2-3x+2)\cdot 3}{(3x+1)^2} = \frac{6x^2\cancel{-9x}+2x-3-3x^2\cancel{+9x}-6}{(3x+1)^2} = \frac{3x^2+2x-9}{(3x+1)^2} \geq 0$$

NUM:

$3x^2+2x-9$

$\Delta = 4-4(-27) = 4(1+27) = 4(28) = 16\cdot 7$

$\sqrt{\Delta} = \sqrt{16}\cdot\sqrt{7} = 4\sqrt{7}$

$$x_{1,2} = \frac{-\cancel{2}^{-1} \pm \cancel{4}^{2}\sqrt{7}}{\cancel{6}_{3}}$$

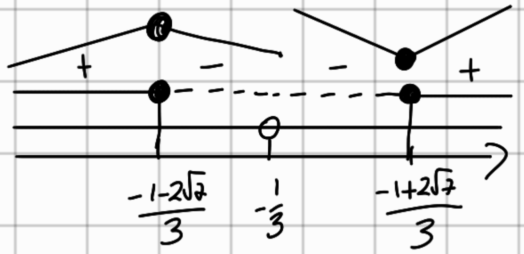
$x_1 = \dfrac{-1-2\sqrt{7}}{3} \simeq -2.09$

$x_2 = \dfrac{-1+2\sqrt{7}}{3} \simeq 1.43$

DEN

$(3x+1)^2 > 0$

$\forall x \in D$



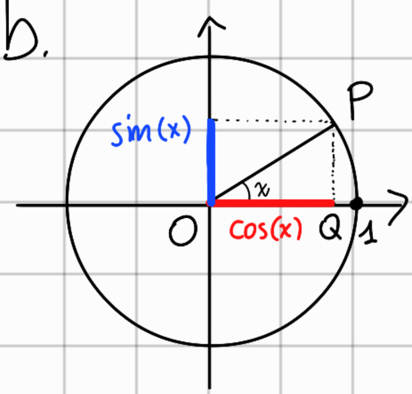
f è CRESCENTE per $x \in \left(-\infty, \frac{-1-2\sqrt{7}}{3}\right) \cup \left(\frac{-1+2\sqrt{7}}{3}, +\infty\right)$

f è DECRESCENTE per $x \in \left(\frac{-1-2\sqrt{7}}{3}, -\frac{1}{3}\right) \cup \left(-\frac{1}{3}, \frac{-1+2\sqrt{7}}{3}\right)$

f è STAZIONARIA per $x = \frac{-1-2\sqrt{7}}{3} \vee x = \frac{-1+2\sqrt{7}}{3}$

$M \simeq (-2.09, -2.39)$, $m \simeq (1.43, -0.04)$

b.

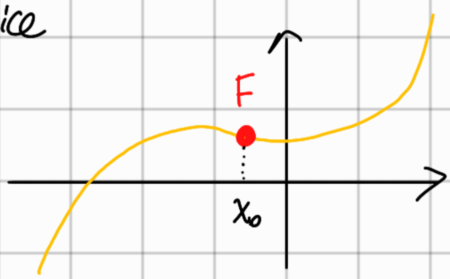


L'equazione fondamentale della trigonometria è

$$\sin^2 x + \cos^2 x = 1$$

Deriva dal Teorema di Pitagora applicato al triangolo rettangolo OPQ : $\overline{PQ}^2 + \overline{OQ}^2 = \overline{OP}^2$

c. Data $f: D \to \mathbb{R}$ funzione, un punto $x_0 \in D$ si dice p.to di FLESSO se $f(x)$ è CONCAVA prima di x_0 e CONVESSA dopo o viceversa.



ESERCIZIO 2

$$\int f'g = fg - \int fg'$$

$$f(x) = (x-\pi)\sin(3x)$$

a. $$\int \underbrace{(x-\pi)}_{g}\underbrace{\sin(3x)}_{f'}dx = -\frac{\cos(3x)}{3}(x-\pi) - \int\left(-\frac{\cos(3x)}{3}\right)\cdot 1\, dx = \frac{1}{3}(\pi-x)\cos(3x) + \frac{1}{3}\frac{\sin(3x)}{3} + C$$

$g' = 1$ $\qquad f = -\frac{\cos(3x)}{3}$

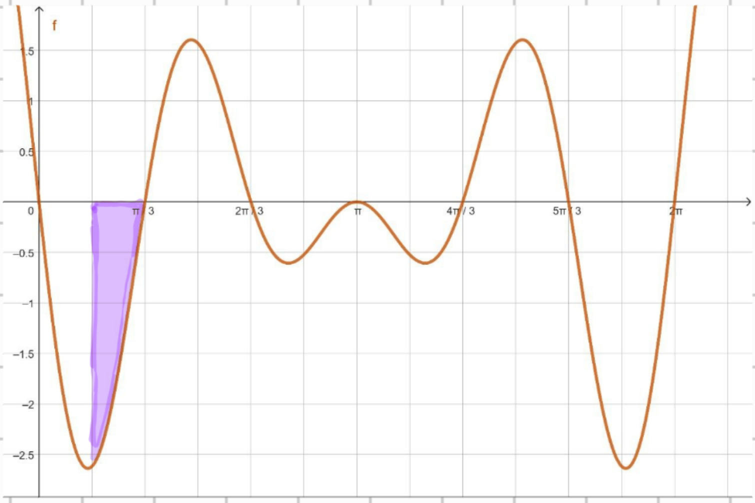
$$= \frac{1}{3}(\pi-x)\cos(3x) + \frac{1}{9}\sin(3x) + C$$

b. $$\int_{\pi/3}^{\pi/6}(x-\pi)\sin(3x)dx = \left[\frac{1}{3}(\pi-x)\cos(3x) + \frac{1}{9}\sin(3x)\right]_{\pi/6}^{\pi/9} =$$

$$= \frac{1}{3}\left(\pi-\frac{\pi}{3}\right)\underbrace{\cos\left(3\frac{\pi}{3}\right)}_{\cos(\pi) = -1} + \cancel{\frac{1}{9}\underbrace{\sin\left(3\frac{\pi}{3}\right)}_{\sin(\pi) = 0}} + \cancel{\frac{1}{3}\left(\pi-\frac{\pi}{3}\right)\underbrace{\cos\left(3\frac{\pi}{6}\right)}_{\cos\left(\frac{\pi}{2}\right) = 0}} - \frac{1}{9}\underbrace{\sin\left(3\frac{\pi}{6}\right)}_{\sin\left(\frac{\pi}{2}\right) = 1} = -\frac{1}{3}\frac{2}{3}\pi - \frac{1}{9} =$$

$$= -\frac{2\pi-1}{9}$$

c.



ESERCIZIO 3

$$\int_0^5 t e^{\left(1-\frac{t^2}{4}\right)}dt = -2\int_0^5 \underbrace{\left(-\frac{1}{2}t\right)}_{g'(t)} e^{\overbrace{\left(1-\frac{t^2}{4}\right)}^{g(t)}}dt = -2\left[e^{\left(1-\frac{t^2}{4}\right)}\right]_0^5 = -2\left[e^{\left(1-\frac{25}{4}\right)} - e^1\right] =$$

$$= -2\left[e^{-21/4} - e\right] = 2e - 2e^{-21/4} \simeq 5.43$$

ESERCIZIO 4

a. Per studiare la relazione tra le due variabili quantitative X "quantità di luce" e Y "altezza pianta" utilizzo il modello della retta di regressione lineare. $y = \alpha X + \beta$ $\boxed{\alpha = \frac{\sigma_{XY}}{\sigma_X^2}}$ $\boxed{\beta = \bar{y} - \alpha\bar{x}}$

x_i	y_i	$(x_i - \bar{x})$	$(x_i-\bar{x})^2$	$(y_i-\bar{y})$	$(x_i-\bar{x})(y_i-\bar{y})$
2	2.3	-4	16	-5.6	22.4
4	5.6	-2	4	-2.3	4.6
6	8.4	0	0	0.5	0
8	10.3	+2	4	2.4	4.8
10	12.9	+4	16	5	20
6 ↓ $\bar{x}$	7.9 ↓ $\bar{y}$				

$$\bar{x} = \frac{1}{M}\sum_{i=1}^{M} x_i = \frac{2+4+6+8+10}{5} = 8 \rightarrow \sigma_X^2 = \frac{1}{M}\sum_{i=1}^{M}(x_i-\bar{x})^2 = \frac{16+4+0+4+16}{5} = 8$$

$$\bar{y} = \frac{2.3+5.6+8.4+10.3+12.9}{5} = 7.9 \qquad \sigma_{XY} = \frac{1}{M}\sum_{i=1}^{M}(x_i-\bar{x})(y_i-\bar{y}) = \frac{1}{5}\left[(-4)(-5.6)+(-2)(-2.3)+(0)(0.5)+(2)(4.8)+(4)(20)\right]$$

$$= \frac{22.4+4.6+0+4.8+20}{5} = 10.36$$

$$\boxed{\alpha = \frac{\sigma_{XY}}{\sigma_X^2} = \frac{10.36}{8} = 1.295}, \quad \boxed{\beta = \bar{y} - \alpha\bar{x} = 7.9 - (1.295)(6) = 0.13}$$

La retta di regressione è $y = (1.295)X + 0.13$

Secondo tale modello se $x^* = 7 \Rightarrow$ $\boxed{y^* = (1.295)\cdot 7 + 0.13 = 9.125}$

b. • $\bar{y} = 7.9$

• $\text{MEDIANA}(Y) \underset{\uparrow}{=} y^{\left(\frac{M+1}{2}\right)} = y^{\left(\frac{5+1}{2}\right)} = y^{(3)} = 8.4$

M = 5 è DISPARI

DATI RIORDINATI

2.3	5.6	8.4	10.3	12.9
$y^{(1)}$	$y^{(2)}$	$y^{(3)}$	$y^{(4)}$	$y^{(5)}$

• La variabile y è una variabile numerica CONTINUA.
(o quantitativa)